Title: A Model of free-falling dust ball Universe

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Abstract:

According to GR, time progress near a massive object becomes slower. We found that in such place, size of any solid matter shrinks smaller than Schwarzschild metric predicts. Therefore, we introduced a new metric named "metric for matter" which shrinks as time progress slows down. With this "metric for matter", we propose a new cosmological model which can explain mysteries of standard big bang model such as dark energy, dark matter, and flatness problem.

One Sentence Summary:

We introduced a new interpretation of General Relativity using "metric for matter" and found that it can explain cosmological mysteries.

Main Text:

The big bang theory is widely accepted as the standard cosmological model that describes the early development of the universe. But there are a lot of fundamental mysteries left unexplained. Dark energy and dark matter was introduced to compensate the gap between theoretical dynamics of stars and galaxies and observational facts. They are widely accepted by most scientists but no one knows what they are. Another biggest problem of big bang theory is so called "flatness problem". Results of observation tell us that spacetime of current universe is very flat. To achieve this flatness today, density of matter and energy in the universe at the Planck era which is at the beginning of the big bang is required to be exactly the critical value with accuracy of one part in 10^{62} or less (1). We present another way for explaining cosmology.

Size of hydrogen atom.

Suppose there is a spherical thin shell whose radius is R, mass is M, Schwarzschild radius is a as shown in Fig. 1B and 1D. The geometry outside the shell M (Place 2) is Schwarzschild geometry whose line element is given by:

$$ds^2 = -\gamma^{-2}(cdt)^2 + \gamma^2 dr^2 + (rd\Omega)^2 \tag{1}$$

$$\gamma = u^t = \frac{dt}{d\tau} = \left(1 - \frac{a}{r}\right)^{-1/2} \tag{2}$$

Using γ defined by eq. 2, proper time $d\tau$ and proper radial distance $d\sigma$ in the Lagrangian coordinates, i.e. coordinates which is fixed to the matter and move with it, are given by:

$$d\tau = dt/\gamma \tag{3}$$

$$d\sigma = \gamma dr \tag{4}$$

According to the Birkhoff's theorem, gravitational acceleration exerted by the mass of spherical shell vanishes inside it and spacetime becomes Minkowski's (2). Considering continuity of the metric toward circumferential direction, the ratio of proper distance and Eulerean distance must be 1 regardless of the direction. Therefore, we can employ Cartesian coordinates of *t, x, y, z* for Eulerean coordinates and *τ, X, Y, Z* for Lagrangian coordinates at place 3 shown in Fig. 1B. Conventional line element ds inside the shell is given by

$$\begin{aligned} ds^2 &= -(cd\tau)^2 + dX^2 + dY^2 + dZ^2 \\ &= -\gamma^{-2}(cdt)^2 + (dx^2 + dy^2 + dz^2) \end{aligned} \qquad (5)$$

Usually, papers concerning GR are written in abstract mathematical manner. But putting aside mathematical elegance, we give priority to intuitive understanding. So, we look into some concrete example. For instance, we work with γ=2.0 world. With eq. 2, γ value at the surface of the shell becomes 2.0 when R=4/3a, and considering the continuity, γ value used in eq.5 must become 2.0 everywhere inside the shell regardless of the direction. There, we calculate the diameter of a hydrogen atom D with Bohr model. Symbols with subscript "1" and "3" denote place 1 and 3, "E" and "L" denote physical value defined with Eulerean and Lagrangian coordinates respectively.

$$c_{3L} = dX / d\tau = c_0 \qquad (6)$$

$$c_{3E} = dx / dt = \frac{1}{\gamma} c_0 \qquad (7)$$

Where c_0 denotes speed of light at infinity. By the principle of equivalence, de Broglie's wavelength λ at place 1 and 3 must be the same in Lagrangian coordinates.

$$\lambda_{3L} = \lambda_{1L} \qquad (8)$$

Then, in Eulerean coordinates,

$$\lambda_{3E} = \frac{1}{\gamma} \lambda_{1E} \qquad (9)$$

$$\frac{D_{3E}}{D_{1E}} = \frac{\lambda_{3E}}{\lambda_{1E}} = \frac{1}{\gamma} \qquad (10)$$

Eq. 10 shows that size of a hydrogen atom in γ=2.0 world becomes half of its original size in Eulerean coordinates. Deductively, if we put a copy of the earth in place 3 in Fig. 1B, speed of a ball thrown by a pitcher becomes half observed in Eulerean coordinates as the speed of light becomes half. But at the same time, as the size of a hydrogen atom shrinks, distance from pitcher's mound to the catcher becomes half of its original. Therefore, flying time of the ball keep the same observed both with Eulerean and Lagrangian coordinates. It is often said that time freezes near the surface of a black hole, but with this thought experiment, time for any matter proceeds at the same speed regardless of γ value. Considering above stated phenomenon, we propose a new line element "material line element" in place of eq. 5 as follows.

$$ds^2 = -(cdt)^2 + \gamma^2(dx^2 + dy^2 + dz^2) \qquad (11)$$

For Schwarzschild geometry, "material line element" is described as follows in place of eq. 1.

$$ds^2 = -(cdt)^2 + \gamma^4 dr^2 + \gamma^2 (rd\Omega)^2 \quad (12)$$

We introduced a new set of metric “metric for matter” defined as coefficients of eq. 11, 12.

In GR, length of time and space are expressed with a set of value and metric. If you change the metric, corresponding value changes but they express the same thing. Therefore, you can use whatever metric you like. For dealing with matter which shrinks as stated above, metric for matter is convenient. For dealing with light, conventional metric is convenient. Thereby, we call it “metric for light” in this study to distinguish them explicitly.

Thought experiment 1: Light and hydrogen atom in γ=2.0 world.

We carried out a thought experiment and expressed the result with Eulerean coordinates and Lagrangian coordinates. With Eulerean coordinates, Minkowski's metric is used. And with Lagrangian coordinates, metric for light and metric for matter are used to express the same physical meaning.

Suppose we have a spherical shell whose mass is *M*, Schwarzschild radius is *a*, and radius *R* is *(4/3)a* as illustrated in Fig. 1B. γ value at the surface of the shell M (place 2) and inside the shell M (place 3) is 2 by eq. 2. From infinitely far place (place 1), green light of wave length λ_G=500nm was emitted for $\Delta\tau$=1 second and irradiated place 2 radially and circumferentially. Beat number of the green light N_G in the duration time is not changed. By eq. 5, duration time of the light at place 2 measured with metric for light becomes a half of its original. Likewise, other value for each metric are calculated and shown in Table 1.

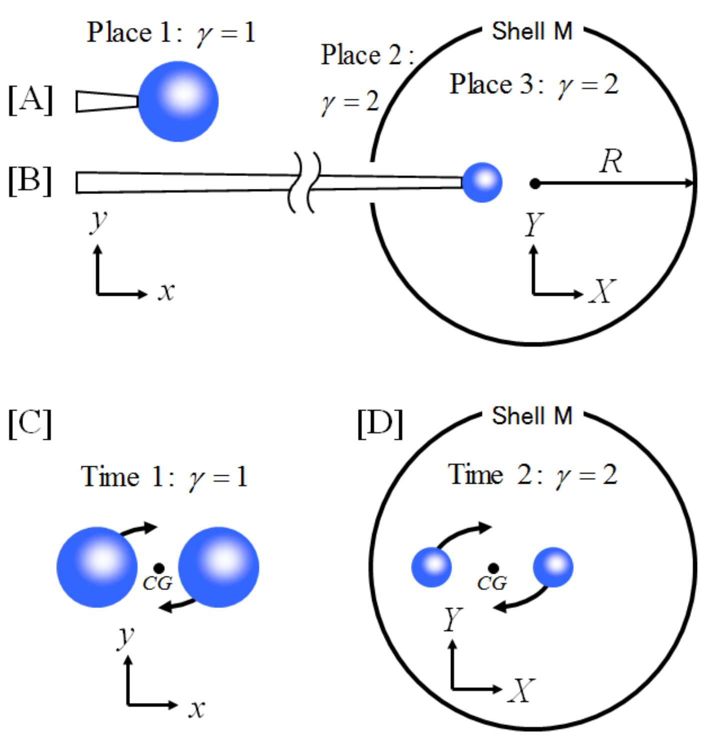


Fig. 1. Settings for thought experiments. (A) Pushing a ball of 1kg with a stick at infinitely far place from the Shell M (place 1). (B) Pushing the same ball in the shell M (place 3) with very long stick from place 1. Place 2 is near the outer surface of the shell M. (C) Two balls of 1 kg are orbiting around the common CG inside shell M (not illustrated) when *R* is infinitely large. (D) The same orbiting balls when the shell shrink to *R= (4/3)a*.

	Place 1	Place 2					
		Radial direction			Circumferential direction		
	Eulerean	Eulerean	Metric (Light)	Metric (Matter)	Eulerean	Metric (Light)	Metric (Matter)
Duration time Δt (S)	1	1	0.5	1	1	0.5	1
Propagation distance L ($\times 10^8$m)	3	0.75	1.5	3	1.5	1.5	3
Speed of light $c=L/\Delta t$ ($\times 10^8$m)	3	0.75	3	3	1.5	3	3
Wave Number N_G ($\times 10^{14}$)	6	6	6	6	6	6	6
Wave length $\lambda_G=L/N_G$ (nm)	500	125	250	500	250	250	500
Diameter of Hydrogen atom D (pm)	53	13.2	26.5	53	26.5	26.5	53

Table 1. Result of thought experiment 1. Green light of wave length λ_G=500nm is emitted from place 1 in Fig. 1A for 1 second and irradiate place 2 from two direction.

Wave length λ_G measured with metric for light becomes shorter in place 2. This phenomenon is called gravitational blue shift. Diameter of a hydrogen atom D becomes smaller in accordance with λ_G, too. But measured with metric for matter, all of the values keep the same as original.

Thought experiment 2: Pushing balls.

Suppose we push two same balls of 1kg with sticks as shown in Fig. 1A and 1B. The size of the ball in place 3 is a half of its original viewed with Eulerean eyes. We are at the end of sticks in place 1 and push the end of the sticks for 1 second. Initial velocity v of the input end is 0 m/S and acceleration rate α is 1 m/S^2. For pushing the ball in place 3, we employ two kinds of very long rigid virtual sticks. First one is a "light metric stick" whose length varies with metric for light. Length of the stick at place 2 shrinks to a half. But the movement distance ΔX of the tip at place 3 keeps the same with the input movement ΔX_0 because metric for length are the same at both ends. Duration time of acceleration at place 3 expressed with each metric are shown on the first line of the Table 2A. Each physical value like velocity and acceleration are calculated from duration time and displacement on the same column. Comparing the leftmost line and rightmost line in table 2A, values do not match. This table shows that the principle of equivalence is not satisfied if we employ "light metric stick".

The other kind of stick is a "material stick" whose length varies with metric for matter. In this case, The movement distance of the tip is a half of the input movement. Physical values at each place are shown in Table 2B. The bottom line of Table 2B shows that the energy at both ends matches right. And comparing the leftmost column and rightmost column, values match perfectly. This shows that the principle of equivalence is satisfied if we employ "material stick".

[A]	Place 1	Place 3		
	Eulerean	Eulerean	Metric (Light)	Metric (matter)
Duration time (S)	1	1	1/2	1
Final displacement ΔX (m)	1/2	1/2	1/2	1
Final velocity $v_f = dX/d\tau$ (m/S)	1	1	2	2
Acceleration α (m/S^2)	1	1	4	2
Four-velocity u^t =dt/dτ (m/s)	c	2c	c	c
Inertial mass $m = m_0 (u^t /c)^2$ (kg)	1	4	1	1
Force $F = m\alpha$ (N)	1	4	4	2
Energy $E = F\Delta X$ (J)	1/2	2	2	2

[B]	Place 1	Place 3		
	Eulerean	Eulerean	Metric (Light)	Metric (matter)
Duration time (S)	1	1	1/2	1
Final displacement ΔX (m)	1/2	1/4	1/4	1/2
Final velocity $v_f = dX/d\tau$ (m/S)	1	1/2	1	1
Acceleration α (m/S^2)	1	1/2	2	1
Four speed $u^t = dt/d\tau$ (m/s)	c	2c	c	c
Inertial mass $m = m_0 (u^t /c)^2$ (kg)	1	4	1	1
Force $F = m\alpha$ (N)	1	2	2	1
Energy $E = F\Delta X$ (J)	1/2	1/2	1/2	1/2

Table 2. Result of thought experiment 2. (A) A ball is pushed with a “light metric stick”. (B) A ball is pushed with a “material stick”.

Thought experiment 3: Falling balls.

Suppose we have two balls of 1 kg at place 3 in Fig. 1B. Initial distance d between them was 2 m and initial velocity of both balls was 0 m/S. They began falling to each other by their own gravity and fell for 1 second. Results are shown in Table 3. In place 3, proportion of $\Delta\tau$ for each metric is the same as the first line in Table 2A and 2B. In place 1, final displacement ΔX_0 is calculated by equation of motion, where symbols with subscript “0” denote value at infinity and G_0 denotes gravitational constant at infinity. Because the speed of time progress slows to a half at place 3, ΔX must be a quarter of original measured with Eulerean and metric for light. Measured with metric for matter, this value doubles because the observer shrinks to a half. Gravitational acceleration α_G is derived from ΔX and $\Delta\tau$ for each metric on the same column. Gravitational mass m is heavier with Eulerean coordinates because it is a function of u^t . Gravitational constant G for each column is derived from m, d, α_G for each column. At place 3, G stays constant with metric for light, but, expressed with metric for matter, it doubles. This shows that with this hypothesis, gravitational constant G grows larger as time go on for an observer who is shrinking in the shrinking shell M.

	Place 1	Place 3		
	Eulerean	Eulerean	Metric (Light)	Metric (matter)
Elapsed time $\Delta\tau$ (S)	1	1	$1/2$	1
Distance d (m)	2	2	2	4
Final displacement ΔX (m)	$\Delta X_0 = 1/8G_0$	$1/4\Delta X_0$	$1/4\Delta X_0$	$1/2\Delta X_0$
Gravitational acceleration α_G (m/S^2)	$\alpha_{G0} = 1/4G_0$	$1/4\ \alpha_{G0}$	α_{G0}	$1/2\ \alpha_{G0}$
Gravitational mass $m=m_0(u^t/c)^2$ (kg)	1	4	1	1
Gravitational constant $G=\alpha_G d^2/m$ (m^3/kgS^2)	G_0	$1/16G_0$	G_0	$2G_0$

Table 3. Result of thought experiment 3. Two balls are falling to each other in the shell M shown in Fig. 1B.

Thought experiment 4: Orbiting balls.

Suppose we have two balls of 1 kg orbiting around the common center of gravity (CG) inside shell M (not illustrated) as shown in Fig. 1C. Radius of the shell was infinitely large and orbiting radius was 1 m at time 1. Because R was large enough, γ value could be assumed to be 1. At time 2, the radius of the shell shrinks to $R= 4/3a$ and γ value inside the shell becomes 2 as shown in Fig. 1D. Position is conserved because of the law of inertia. Kinetic energy T is conserved because of the energy conservation law. From T, m, $\Delta\tau$, circling velocity v and centrifugal acceleration α_c for each column is obtained. If centrifugal acceleration was balanced with gravitational acceleration at time 1, α_G/α_c value on all the columns become 1. Right three columns express the same meaning with different metric. Therefore, value of dimensionless number like α_G/α_c must be the same. And in GR, energy is called a “scalar physical quantity” which must not vary with coordinate transformation. In Table 4, values of calculated potential energy V on all columns keep invariant as a scalar. Different from energy, the value of gravitational constant G depends on metric. Most scientist who is concerned with GR use geometrized unit system where c = 1, G = 1. This must have numbed the sense for G constant. We consider that confusion on this point is the root cause of the energy conservation disputes with GR. And this is the reason we put physical units after all variables here.

According to Table 3, orbiting objects keep its orbiting radius with metric for light. For an observer who lives on a copy of the earth in the shell M of Fig. 1D, the moon is observed twice as high as time 1. This shows that growing G constant itself does not explain dark matter.

But there are reasons by which we consider dark matter can be explained with this model. At first, dark matter was predicted to explain galaxy rotation curve and concentration. If stars in a galaxy are rotating steadily, rotating velocity near the center must be faster than outside, but the speed is found almost the same at all the region. And observed total mass is much lighter than necessary mass needed for pulling stars together. With Newtonian dynamics, if stars are pulled together by their own gravity, stars gain kinetic energy and with it, they fly away to the original distance. Some kind of damping is required to kill the kinetic energy and keep them together. In the scale of solar system, physical contact between matters can do the work. But in the scale of galaxies, it that is not likely because matters are so rare to have physical contact. We considered that they are on their first way heading to the gravity source. Suppose a star is floating still at very far place from a galaxy inside a huge scale shell M. The star gains velocity by gravitational pull. By the time the star goes half way, what if the shell shrink and γ value double? The distance is evaluated as far as initial distance, but the star has got certain velocity. This effect can make dynamics different from Newtonian and if the scale is larger, the effect is larger.

	Time 1	Time 2		
	Eulerean	Eulerean	Metric (Light)	Metric (matter)
Circling radius r (m)	1	1	1	2
Kinetic energy $T=mv^2/2$ (J)	T_0	T_0	T_0	T_0
Circling velocity $v=dX/d\tau$ (m/s)	v_0	$1/2v_0$	v_0	v_0
Centrifugal acceleration α_c (m/S^2)	$\alpha_{c0} = -v_0^2$	$1/4\,\alpha_{c0}$	α_{c0}	$1/2\,\alpha_{c0}$
α_G/α_c	1	1	1	1
Potential energy $V=-Gm^2/(4r)$ (J)	$-1/4G_0$	$-1/4G_0$	$-1/4G_0$	$-1/4G_0$

Table 4. Result of thought experiment 4. Two balls are circling around the common CG.

Genesis scenario of conversing universe.

Argument on genesis scenario can be out of physics because no one can prove it. But to justify the purpose of this study, we propose a new scenario. According to most accepted big bang theory, our universe was created from vacuum by quantum mechanism. It began as a small particle and expanded. Initial smallness can explain homogeneity of current universe because it was small enough to mix up. But, expansion stage is controversial. There is a big question so called “flatness problem”. Results of observation tell us that spacetime of current universe is very flat. To achieve this flatness today, density of matter and energy in the universe at the Planck era i.e. at the beginning of the big bang, is required to be exactly the critical value with accuracy of one part in 10^{62} or less.

In our hypothesis, we adopt first half of big bang theory. The universe was born as a small particle by quantum mechanism and small enough to be homogeneous. But how you could know it was small when there was no scale to measure. Cosmological scale is ultimately defined by balance of physical forces and time. If the physical rules are created at this point, it could be anything. Measuring the new-born particle with newly created scale, it could turn out to be billion or trillion or whatever light years across. This hypothesis seems to be stupendous. But, because it is only the change of scale, the expansion process is calm and quiet event in a moment. Compared with the fierceness of the inflation process, stupendousness of this hypothesis is not larger.

In standard model, structure of the universe is explained with Robertson–Walker (RW) model (3). It states that the universe has limited volume but no center nor outer edge. This model could be mathematically viable, but physically, no one can imagine any concrete shape which satisfies such conditions. In our hypothesis, we adopted much simpler dust ball universe model. The expanded spherical dust ball began to free-fall toward its center of gravity at the time of creation. Dust particles which represent stars and galaxies became congested at first, but with strong relativistic effect, as following numerical simulation shows, it began to get spacious from certain time point.

Numerical simulation: Free-falling dust ball universe.

We hit upon the idea of free-falling dust ball universe a couple of years ago and posted it on Science in June 2011. We named it as "Micro Bang". But it lacked theoretical argumentation and deservedly, it was rejected. Since then, we developed two generations of original code for simulating radius-time two dimensional free-falling dust ball universe. First half of this study was made to clarify the basics. First code employed rather orthodox method using geodesic equations. It worked well under weakly relativistic situation. But as we were afraid of, it blew up under strongly relativistic situation. We tried various techniques to suppress the divergence, but the effort was in vain. After a long struggle, we hit upon an idea for second generation simulation code. We abandoned geodesic equations which are differential formulation, and instead, employed integral formulation for equations of motion.

$$\frac{du^r}{d\tau} = \frac{d^2 r}{d\tau^2} = -G\frac{m(\sigma)}{r^2} \qquad (13)$$

$$u^r{}_{(t+\Delta t,\sigma)} = u^r{}_{(t,\sigma)} + \frac{du^r}{d\tau}\Delta\tau \qquad (14)$$

$$r_{(t+\Delta t,\sigma)} = r_{(t,\sigma)} + u^r{}_{(t,\sigma)}\Delta\tau \qquad (15)$$

$$\phi_{(\Sigma)} = -\frac{Gm(\Sigma)}{R} \qquad (16)$$

$$\phi_{(\sigma)} = \phi(\sigma+\Delta\sigma) + \frac{du^r}{d\tau}\Delta r \qquad (17)$$

$$\gamma = u^t = \frac{dt}{d\tau} = \frac{1}{\sqrt{1+2\phi}} \qquad (18)$$

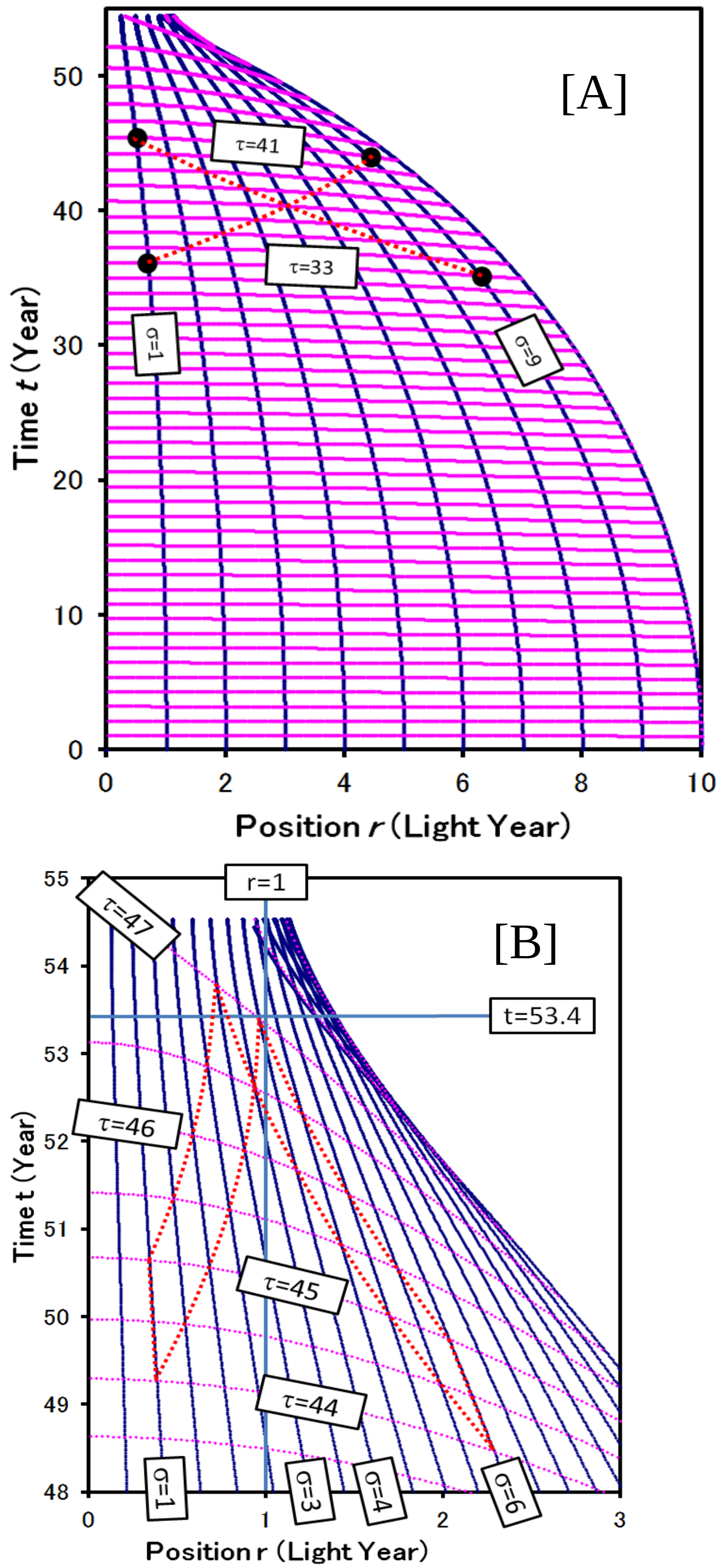
Where σ, Σ denote radial position and radius of the dust ball in Lagrangian coordinates, m(σ), ϕ (σ) denote mass of the dust inside radius σ, gravitational potential at radius σ. Gravitational potential $\phi(\Sigma)$ and γ value at the surface are obtained by Schwarzschild geometry. Those values inside the dust ball are obtained by eq. 17, 18. Equation of motion corresponding with geodesic equation is eq.13. From moving observer, gravitational mass of dust particles is observed heavier. But in this case, all particles stay still with Lagrangian coordinates. Therefore, from radius σ, mass of the dust inside radius σ is observed to be m(σ) throughout the time with metric for light. This is the basis of eq.13.

This code turned out to be robust but still blew up from the outermost shell element of the dust ball. After another long struggle, we hit upon a thought experiment. Suppose we have a dust shell of mass *M* which has small finite thickness. Outermost dust particles are accelerated but innermost dust particles are floating in zero gravity. Therefore, by natural providence, outermost particles overtake next outermost particles. For this reason, we changed our code to allow dust shell elements to overtake inner shell elements. By this amendment, our code became robust enough to simulate dynamics under extremely strong relativistic conditions.

For instance, we simulated free-falling homogeneous dust ball whose Schwarzschild radius *a* is one light year. Initial radius *R* is10 light years, initial radial velocity u^r is *0*. The result is shown in Fig. 2A. Horizontal pink curves represent simultaneous proper time lines where proper time τ is constant. Vertical blue curves represent geodesics where proper position σ is constant. With GR, in general, proper time and proper distance are defined locally at that particular point. But it is known that if everything is free-falling and stays still in Lagrangian coordinates, proper time and proper space are shared by all dust particles (4).

In Fig. 2A, at time $t = \tau = 0$, σ is identical with *r* and begin falling. Distance Δr between dust particles at σ = 1 and σ = 9 contracts in Eulerean coordinates. But with Lagrangian coordinates, $\Delta\sigma$ keeps the same value of 8 light years throughout the time. Suppose light was emitted from dust at σ = 1 and σ = 9 at their shared proper time τ = 33. The path of the light is represented by red dotted curves. They propagate from grid point to grid point because they propagate one local light year $\Delta\sigma$ in one local year $\Delta\tau$. Therefore, they reach to the other dust particle after $\Delta\tau = 8$ years for both particles simultaneously. This shows that they share the same proper time and proper space. And for an observer who is free-falling with dust particles, the space is completely flat. This can explain the flatness of our current universe.

Each dust particle describes a parabolic line at first. But when the ball’s radius R approaches Schwarzschild radius, falling velocity in Eulerean coordinates slows down because time progress slows. And because this effect is stronger at inner place, dust particle of outer region catch up with inner dust particle and mass are condensed to form a high density shell at outermost region. This is similar to the situation described in Fig. 1D.


[A]
τ=41
τ=33
σ=1
σ=9
Time t (Year)
Position r (Light Year)
[B]
r=1
τ=47
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τ=46
τ=45
τ=44
σ=1
σ=3
σ=4
σ=6
Time t (Year)
Position r (Light Year)

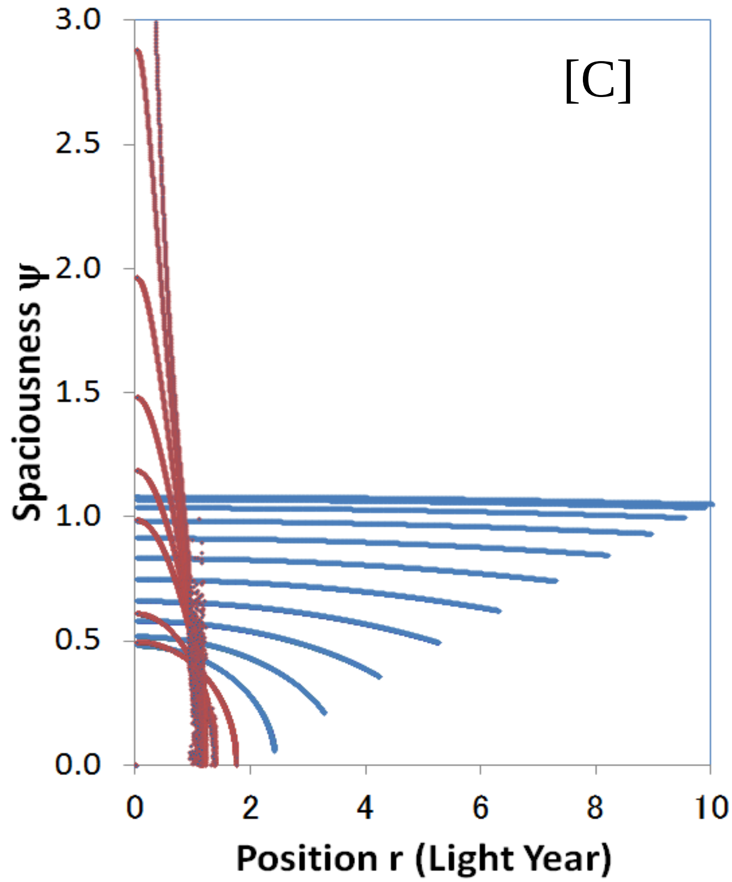

Fig. 2. Simulation results. (A) Homogeneous spherical dust ball of Schwarzschild radius 1 light year, initial radius 10 light years began free-falling at t = t = 0. (B) Close-up of Fig. 2A. (C) Transition of spaciousness Ψ of the same simulation. Time interval between each curve is not constant.

Fig. 2B is a close-up of the last part of Fig. 2A. Suppose green light of wave length λ_G = 500 nm was emitted from dust particles at σ = 1 light year and σ = 6 light year from shared proper time of τ = 44 to 45 year toward the other dust particle. Length of light arrows become 1 light year from both particles when τ = 45 year. Total wave number $N_G = c\Delta\tau/\lambda_G$ is 1.9×10^{22} each. As the front end and rear end of the light arrows propagate one local light year $\Delta\sigma$ in one local year $\Delta\tau$, they pass through grid point to grid point. For a stationary observer at time t = 53.4 year, simultaneous time line is parallel to the horizontal axis. Length of light arrow emitted from outer particle is shorter than one light year. But within that length, all wave number N_G is packed in. This is observed as gravitational blue shift. And length of the light arrow from the inner particle is observed longer than one light year. This is gravitational red shift.

Both light arrows reach between σ = 3 light year and σ = 4 light year at τ = 47 year. There, length of both light arrows is exactly one local light year and N_G keeps the same. Therefore, in Lagrangian coordinates, wave length measured with metric for light keeps the same value regardless of the direction the light came. But by this time, size of the observer who is free-falling with dust shrinks because γ value at τ = 47 year is larger than before. Consequently, for this observer, wave length of the light coming from both places are observed longer than its original observed with metric for matter which is related to the size of the observer. This phenomenon can explain cosmological red shift and dark energy. And the expansion accelerates as shown in Fig. 2C.

Strictly speaking, light came from inner place has shorter wave length than light from outer place with this hypothesis because objects at inner place shrink earlier. But this difference can be small enough if the original dust ball was large enough or the observer is not far from the center of the dust ball. And actually cosmic microwave background radiation (CMB) is a little anisotropic. This anisotropy is interpreted as Doppler shift caused by our traveling speed against CMB. But it is possible to interpret it as indicator of the direction of the center of the universe.

Measured with metric for matter, at place where an observer shrinks, space between dust particles look wider. Therefore, we defined spaciousness Ψ as follows.

$$\Psi = \gamma \frac{\Delta\sigma}{\Delta\sigma_i} \qquad (19)$$

Where $\Delta\sigma_i$ denotes initial radial distance of arbitrary two adjacent dust particles. In Fig. 2C, transition of the distribution of spaciousness Ψ of the same simulation is shown. Each curve represents distribution of Ψ at each proper time τ. Time interval between each curve is not constant. At t = τ = 0, Ψ curve spreads from r = 0 to r = 10 light year because the initial radius of the dust ball is 10 light years. After the release, radius of the ball shrinks. If this was a Newtonian simulation, the intensity of gravitational acceleration is proportional to the position r. Consequently, it shrinks keeping the mass distribution similar to its original. And all of dust particles converge to r = 0 to form a singular point. But in this simulation, time proceeds faster at outer region. Therefore, dust particles at outer part approach to inner particles. This makes outer part less spacious. After a while, outermost dust particles catch up with next outermost dust particles. At this point, Ψ at the outermost dust particle becomes 0. And after this, dust particles of outer region begin to be accumulated around the outermost dust particle and form a shell like structure. This makes a similar situation as Fig. 1D. As the shell like structure approaches to Schwarzschild radius, γ value around the center increases. At the last part of this simulation, contracting velocity declines because of the slow time progress and growth speed of γ increases. Consequently, spaciousness Ψ increases rapidly. Distribution of Ψ after this time is plotted in red mark. In this simulation, Ψ increased to as large as thousands at the center before one variable in the program overflowed its double precision number limit. It is often said that in black hole, matters fall into a singular point where the density is infinite. But according to this simulation, we could find nothing special around the center. To the contrary, the center of dust ball becomes more and more spacious like our current universe. This is not a mathematical proof, but it is unlikely that black hole is formed within a finite time span. In other word, it is likely that it will not be formed forever.

Extreme γ value is a hurdle for simulation, but it is not seem to be a hurdle to stop the time progress of real world. It is not sufficient to decide whether time t in Eulerean coordinates continues forever without having the big crunch or not. But at least, time measured in metric for matter seems to last forever because people living inside can become infinitely small. We will have big rip with this hypothesis. Analogous phenomena must happen in atomic scale and quantum scale.

References and Notes:

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